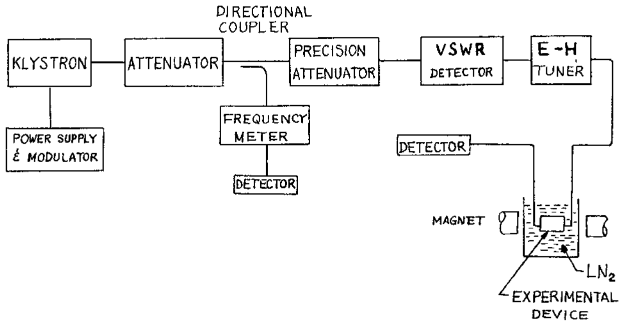

Fig. 2. Experimental set-up for the measurement of characteristics of the experimental microwave device.

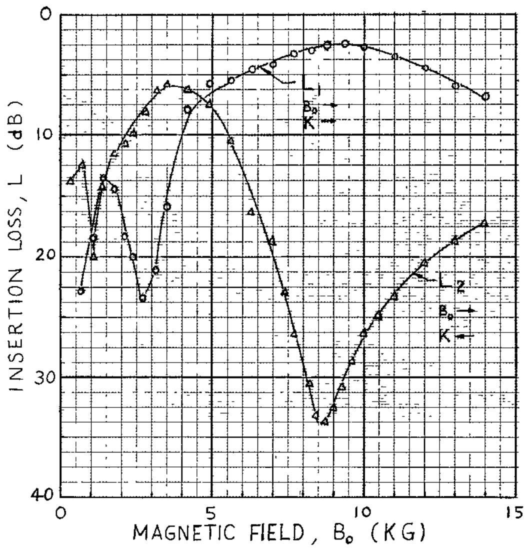

Fig. 3. Measured nonreciprocal transmission characteristics of the experimental microwave device as a function of magnetic field.

lent isolation ratio was achieved. The measured isolation ratio at this point was 30 dB, i.e., the ratio of 1000 to 1 in power between L_1 and L_2.

We have thus demonstrated an example of possible microwave device applications of the Faraday effect in solid-state plasmas. The theoretical upper limit of frequency of the solid-state plasma device will appear at the cyclotron resonant frequency which is higher than 1000 GHz in the present case. Although the present experiment was conducted at a K_a-band frequency, the technique can, therefore, be extended to higher frequency regions such as submillimeter waves.

H. J. Kuno
W. D. Hershberger
Dept. of Engrg.
University of California
Los Angeles, Calif.

Further Generalization of Waveguide Theorems

In a recent paper by Laxpati and Mittra [1], the bidirectional waveguide theorems [2], [3] were extended to both periodic and open waveguide structures. In the original derivation of the theorems [2], [3] an artifice was employed in which Poynting's theorem was applied to a standing complex wave set up in the general bidirectional waveguide by a shorting plane. The point was well taken in the recent paper [1] that a short-circuit termination is not necessary and that the same results may be derived in just as straightforward a manner by considering a termination of arbitrary, non-zero reflection coefficient which also sets up a complex standing wave. The important features of the power and pseudo-energy relations result from the cross terms arising in the complex standing wave and are lost if only a traveling-wave without a reflection is operated on by Poynting's theorem.

In this correspondence we wish to point out that the bidirectional-waveguide theorems may be derived even without the artifice of an imperfect termination. Only the relations for a single wave in an infinite or matched waveguide need be considered [4]. Furthermore, the same basic theorems may be extended to apply to *nonbidirectional* waveguides as well [4]. Also, the constitutive relations characterizing the medium filling the waveguide can be generalized to include the Tellegen medium for which the relations apply [5]

$$\overline{D} = \overline{\overline{\epsilon}}\cdot\overline{E} + \overline{\overline{\gamma}}\cdot\overline{H} \quad (1)$$

$$\overline{B} = \overline{\overline{\mu}}\cdot\overline{H} + \overline{\overline{\xi}}\cdot\overline{E}. \quad (2)$$

The Tellegen medium of course reduces to the more ordinary anisotropic medium when $\overline{\overline{\xi}}$ and $\overline{\overline{\gamma}}$ vanish. In this correspondence we shall confine our attention to the lossless, passive systems. Thus for the Tellegen medium, the following relations must apply [5]

$$\overline{\overline{\epsilon}} = \overline{\overline{\epsilon}}^{+} \qquad \overline{\overline{\mu}} = \overline{\overline{\mu}}^{+} \qquad \overline{\overline{\gamma}} = \overline{\overline{\xi}}^{+} \quad (3)$$

where the superscript (+) has been used to denote the conjugate transpose of a tensor.

For a waveguide containing the general Tellegen medium, Maxwell's equations may be separated into transverse and longitudinal components for a single waveguide mode:

$$\nabla_T E_z + \Gamma \overline{E}_T = -j\omega \bar{i}_z \times \overline{B}_T \quad (4)$$

$$\nabla_T H_z + \Gamma \overline{H}_T = j\omega \bar{i}_z \times \overline{D}_T \quad (5)$$

$$\bar{i}_z \cdot \nabla_T \times \overline{E}_T = -j\omega B_z \quad (6)$$

$$\bar{i}_z \cdot \nabla_T \times \overline{H}_T = j\omega D_z \quad (7)$$

where Γ in general is the complex propagation constant for the mode. The relations leading to the waveguide theorems are formed by cross multiplying (4) by $\overline{H}_T^*$ and then dot multiplying by the unit vector $\bar{i}_z$. The result is

$$\Gamma(\overline{E}_T \times \overline{H}_T^*)\cdot\bar{i}_z = j\omega\overline{H}_T^*\cdot\overline{B}_T - j\omega E_z D_z^* - \bar{i}_z\cdot\nabla_T \times (E_z\overline{H}_T^*) \quad (8)$$

where (7) has also been used. In a similar manner, we have from (5) and (6)

$$\Gamma^*(\overline{E}_T \times \overline{H}_T^*)\cdot\bar{i}_z = -j\omega\overline{E}_T\cdot\overline{D}_T^* + j\omega H_z^* B_z + \bar{i}_z\cdot\nabla_T \times (H_z^*\overline{E}_T). \quad (9)$$

When (8) and (9) are integrated over the cross section of the waveguide, we obtain the relations[1]

$$(\alpha + j\beta)(P + jQ) = j2\omega(U_{mT} - U_{ez}) \quad (10)$$

$$(\alpha - j\beta)(P + jQ) = j2\omega(U_{mz} - U_{eT}). \quad (11)$$

The quantities appearing in these equations have the definitions of

a) complex power

$$P + jQ = \frac{1}{2}\int \overline{E}_T \times \overline{H}_T^*\cdot\bar{i}_z da, \quad (12)$$

b) transverse magnetic pseudo energy

$$U_{mT} = \frac{1}{4}\int \overline{H}_T^*\cdot\overline{B}_T da, \quad (13)$$

c) transverse electric pseudo energy

$$U_{eT} = \frac{1}{4}\int \overline{E}_T\cdot\overline{D}_T^* da, \quad (14)$$

d) longitudinal magnetic pseudo energy

$$U_{mz} = \frac{1}{4}\int H_z^* B_z da, \quad (15)$$

e) longitudinal electric pseudo energy

$$U_{ez} = \frac{1}{4}\int E_z D_z^* da. \quad (16)$$

An important feature to note at this point is that the pseudo energies defined here are in general complex quantities except for the special case of bidirectional waveguides. For bidirectional waveguides, they are all pure real since $\overline{B}_T$ and $\overline{D}_T$ involve only $\overline{H}_T$ and $\overline{E}_T$, respectively, and B_z and D_z involve only H_z and E_z, respectively.

Manuscript received July 22, 1966. This work was supported in part by Joint Services Electronics Program Contract DA-36-039-AMC-03200(E) and National Science Foundation Grant GK-614.

[1] The terms involving the transverse curls vanish by applying Stokes' Theorem and the boundary conditions at perfect electric and/or magnetic walls.

Equations (11) and (12) may be manipulated to yield

$$\alpha(P + jQ) = j\omega(U_m - U_e) \qquad (17)$$

$$\beta(P + jQ) = \omega(U_T - U_z) \qquad (18)$$

in which

$$U_m = U_{mT} + U_{mz} \qquad (19)$$

$$U_e = U_{eT} + U_{ez} \qquad (20)$$

$$U_T = U_{mT} + U_{eT} \qquad (21)$$

$$U_z = U_{mz} + U_{ez}. \qquad (22)$$

The quantities U_m and U_e are in general pure real for lossless, passive systems. However U_T and U_z are in general complex. Thus, the following waveguide theorems result.

a) For propagating waves ($\Gamma = j\beta$)

$$P = \frac{\omega}{\beta} \operatorname{Re}(U_T - U_z)$$

$$Q = \frac{\omega}{\beta} \operatorname{Im}(U_T - U_z)$$

$$U_m = U_e.$$

b) For evanescent waves ($\Gamma = \alpha$)

$$P = 0$$

$$Q = \frac{\omega}{\alpha}(U_m - U_e)$$

$$\operatorname{Re}(U_T) = \operatorname{Re}(U_z)$$

$$\operatorname{Im}(U_T) = \operatorname{Im}(U_z).$$

c) For complex waves ($\Gamma = \alpha + j\beta$)

$$P = 0$$

$$Q = \frac{\omega}{\alpha}(U_m - U_e)$$

$$\operatorname{Re}(U_T) = \operatorname{Re}(U_z)$$

$$\alpha \operatorname{Im}(U_T - U_z) = \beta(U_m - U_e).$$

These theorems apply, in general, to waveguides containing lossless, passive Tellegen media. For the special case of bidirectional waveguides, all of the transverse and longitudinal pseudo energies are pure real and the theorems presented above reduce to the more familiar ones [1]-[4].

In summary, we have demonstrated that the bidirectional waveguide theorems need not be derived by considering a standing wave and can be derived directly from the transverse and longitudinal components of Maxwell's equations for a single waveguide mode. Furthermore, these theorems may be generalized to include nonbidirectional waveguides containing the Tellegen medium. The results for each of the three different types of waves give relations similar to those obtained for bidirectional waveguides. As a consequence of the possibility of complex pseudo-energy terms for nonbidirectional waveguides, the interesting properties are revealed that propagating waves can carry reactive power as well as real power, while evanescent waves and complex waves carry reactive power but no real power.

Because of the general application of these theorems to nonbidirectional waveguides as well as bidirectional, we feel an appropriate name by which they should be referred to is the "waveguide power-mode theorems." The use of the combination "power-mode" stresses that the theorems deal with power relationships for a *single* waveguide mode.

The method of derivation employed here can be used to extend the work of Laxpati and Mittra [1] on periodic and open waveguides, again without employing the artifice of setting up standing waves. Also, waveguide systems that contain active media such as electron beams can also be handled in a similar manner. Active waveguides will be the subject of a future treatment by the authors.

P. Chorney
Microwave Associates, Inc.
Burlington, Mass.
A. Bers
P. Penfield
Dept. of Elec. Engrg.
and Research Lab. of Electronics
Massachusetts Institute of Technology
Cambridge, Mass.

References

[1] S. R. Laxpati and R. Mittra, "Energy considerations in open and closed waveguides," *IEEE Trans. on Antennas and Propagation*, vol. AP-13, pp. 883–890, November 1965.
[2] P. Chorney, "Power and energy relations in bidirectional waveguides," in *Proc. of Symp. on Electromagnetics and Fluid Dynamics of Gaseous Plasma*, New York: Polytechnic Press, 1961, pp. 195–210.
[3] P. Chorney, "Power and energy relations in bidirectional waveguides," Research Lab. of Electronics, Mass. Inst. Tech., Cambridge, Mass., Tech. Rep 396, September 1, 1961.
[4] W. P. Allis, S. J. Buchsbaum, and A. Bers, *Waves in Anisotropic Plasmas*. Cambridge, Mass.: M.I.T. Press, 1963, sec. 7.6.
[5] B. D. H. Tellegen, "The gyrator, a new network element," *Phillips Res. Repts.*, vol. 3, pp. 81–101, April 1948.

Plotting the Electromagnetic Field by Power Dissipation

Useful information on the electromagnetic field traveling inside a waveguide can be obtained by detecting, with temperature sensitive paints, the dissipated power on diaphragms placed transversely to the guide axis.

In the study of some ferrite structures, a technique was used which allowed the experimental verification of some properties of the electromagnetic field by detecting the RF power dissipation on certain characteristic regions [1], [2].

This correspondence deals with an extension of the preceding technique to obtain some useful and straight information about the field traveling inside a waveguide.

When a thin dissipative sheet is placed transversely to the axis of a waveguide, if only one mode is propagating, the dissipated power on the sheet follows exactly the distribution of the Poynting vector, for TE modes. By neglecting the transverse flow of power on the sheet, which is certainly permissible if the conductivity is not too low, it can give, with good approximation, the Poynting vector distribution for TM modes. If more than one mode is propagating inside the guide, while simple inspection of the power dissipation plot does not in general allow a detailed modal analysis, detection of more than one mode is certainly possible.

To reveal the dissipated power, a temperature sensitive paint has been used, which at a specific temperature (in this case 43°C) melts and becomes transparent. In this way the points where the melting of the paint starts, indicate the position of the maxima of the Poynting vector. But if the dissipative diaphragm is realized in such a way that a large

Manuscript received June 30, 1966.

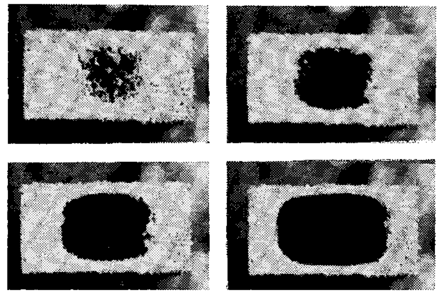

Fig. 1. Dissipation plot on a thin graphited paper placed at the open-terminal section of standard X-band waveguide, fed with a 5 watt klystron at 10 GHz. The photographs were taken at intervals of about 10 seconds.

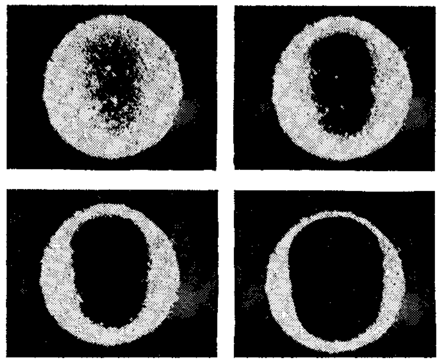

Fig. 2. Dissipation plot on a thin graphited paper placed at the open-terminal section of a circular guide—in which only the fundamental mode can propagate—fed with a 5 watt klystron at 10 GHz. The photographs were taken at intervals of about 10 seconds. The slight asymmetry is probably due to some nonhomogeneity of the graphited sheet.

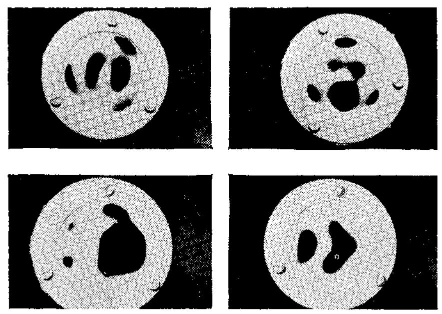

Fig. 3. Typical dissipation plots that can be observed on a thin graphited paper placed at the open-terminal section of a multimode circular guide fed with a 5 watt klystron at 10 GHz for different values of polarization of the exciting field. The inner diameter of the guide is 53 mm.

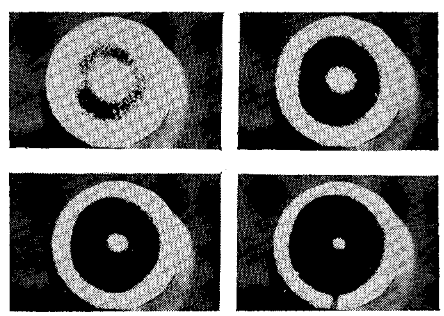

Fig. 4. Dissipation plot on a thin graphited paper placed at the open-terminal section of a circular guide fed with 5 watt klystron at 10 GHz and with the walls made with an isolated copper wire of 0.2 mm of diameter tightly wound. The inner diameter of the guide is 53 mm. The photographs were taken at intervals of about 10 seconds.